Preprint submitted to Elsevier Science1 February 2008

Quantum manipulation and simulation using

Josephson junction arrays

Xingxiang Zhou and Ari Mizel

Department of Physics, The Pennsylvania State University, University Park, PA

16802, USA

Abstract

We discuss the prospect of using quantum properties of large scale Josephson junc-

tion arrays for quantum manipulation and simulation. We study the collective vi-

brational quantum modes of a Josephson junction array and show that they provide

a natural and practical method for realizing a high quality cavity for superconduct-

ing qubit based QED. We further demonstrate that by using Josephson junction

arrays we can simulate a family of problems concerning spinless electron-phonon

and electron-electron interactions. These protocols require no or few controls over

the Josephson junction array and are thus relatively easy to realize given currently

available technology.

Key words: Qubit, Quantum computing, Josephson junction array

PACS: 03.67.Lx, 74.90.+n, 85.25.Dq, 85.25.Cp

Email

ad-

---
dresses:

xizhou@yahoo.com

(Xingx-

---
i-

---
ang

Zhou),

ari@phys.psu.edu

(Ari

Mizel).

Superconducting

de-

vice

based

solid

37

state

qubits

[1]

are

at-

trac-

tive

be-

cause

of

their

in-

her-

ent

51

scal-

a-

53

bil-

ity.

55

Mi-

crowave

spec-

troscopy

and

long

lived

pop-

u-

la-

tion

os-

cil-

la-

tion

con-

sis-

tent

with

sin-

gle

[2,3,4,5,6,7,8]

and

two

qubit

quan-

tum

states

[9,10,11,12]

have

been

ob-

served

ex-

per-

i-

men-

tally.

Recently,

a

new

ap-

proach

to

scal-

able

su-

per-

con-

duct-

ing

quan-

tum

com-

put-

ing

anal-

o-

gous

to

115

atomic

cavity-

QED

was

stud-

ied

the-

o-

ret-

i-

cally

[13,14,15,16]

and

im-

ple-

mented

ex-

per-

i-

men-

tally

[17,18].

This

new

ap-

proach

opens

the

pos-

si-

145

bil-

ity

of

ap-

ply-

ing

meth-

ods

and

prin-

155

ci-

ples

from

the

159

rich

field

of

atomic

QED

in

solid

state

quan-

tum

169

in-

for-

171

ma-

tion

pro-

cess-

ing.

One

prac-

ti-

cal

prob-

lem

of

183

solid

state

185

qubit

based

QED

is

the

re-

al-

iza-

tion

of

a

high

197

qual-

ity

res-

200

onator

201

to

which

203

many

qubits

205

can

cou-

ple.

A

209

lumped

el-

e-

ment

on-

chip

215

LC

cir-

cuit,

such

as

that

221

used

in

[18],

suf-

225

fers

from

di-

elec-

229

tric

230

loss

of

the

ca-

pac-

i-

tor

and

ac

239

loss

of

the

su-

per-

con-

duc-

tor

[19].

A

249

high

qual-

251

ity

res-

onator

can

be

re-

257

al-

ized

with

a

co-

planar

263

waveg-

uide

if

high

qual-

ity

sub-

strate

ma-

te-

273

rial

(such

as

sap-

277

phire)

is

279

used

to

min-

i-

mize

the

285

loss

[20,21].

A

high

qual-

ity,

291

low

leak-

293

age

Joseph-

son

junc-

tion

pro-

vides

a

301

nat-

u-

ral

and

305

easy

re-

307

al-

iza-

tion

of

a

high

qual-

ity

315

res-

onator

due

to

its

high

qual-

ity

tri-

layer

struc-

ture

327

[22],

how-

ever

only

a

few

333

qubits

can

cou-

ple

to

such

a

sin-

341

gle

junc-

tion

res-

onator

[13,14].

In

this

work,

we

351

study

the

quan-

tum

dy-

nam-

357

ics

of

359

a

Joseph-

361

son

junc-

363

tion

ar-

365

ray

and

show

that

the

“phonon

modes”

cor-

re-

spond-

ing

to

the

small

379

col-

lec-

381

tive

vi-

bra-

tions

of

the

junc-

tion

389

phases

can

be

used

to

re-

al-

ize

a

high

399

qual-

ity

401

res-

onator

to

which

405

many

su-

per-

con-

duct-

ing

411

qubits

can

413

cou-

ple.

The

re-

417

sul-

tant

struc-

ture

is

anal-

o-

gous

to

the

427

ion

trap

quan-

tum

com-

puter

433

in

which

435

qubits

com-

mu-

ni-

439

cate

through

441

the

phonon

modes

[23].

We

fur-

ther

show

that,

by

us-

ing

453

a

prop-

455

erly

cou-

pled

su-

per-

con-

duct-

ing

463

qubit

ar-

465

ray

we

can

sim-

u-

late

a

fam-

473

ily

of

prob-

lems

in-

volv-

ing

spin-

481

less

electron-

483

phonon

and

electron-

electron

in-

ter-

ac-

tions.

Consider

the

sim-

ple

Joseph-

son

junc-

tion

ar-

ray

shown

in

503

Fig.

1

(a).

De-

507

note

the

509

phases

across

511

the

ver-

ti-

cal

junc-

tions

517

$\theta_0,$

θ_1,

...,

θ_{N-1}

(the

phase

523

of

the

ground

is

527

set

to

0).

The

ca-

pac-

i-

tance

of

the

ver-

ti-

cal

junc-

tions

is

C.

The

hor-

i-

zon-

tal

junc-

tions

551

are

much

553

big-

ger

in

size

than

the

ver-

ti-

561

cal

ones,

and

their

crit-

i-

567

cal

cur-

rent

is

571

K^2

times

573

that

of

575

the

ver-

ti-

cal

junc-

tions

581

(I_c),

where

$K \gg$

1.

In

prac-

tice,

all

junc-

tions

591

can

be

re-

al-

595

ized

by

597

low

self-

599

inductance

dc-

SQUIDs

to

al-

low

605

tun-

ing

607

of

their

crit-

i-

611

cal

cur-

rents;

there-

fore

K^2

is

not

nec-

es-

sar-

ily

equal

to

the

ra-

tio

of

the

junc-

tion

sizes.

633

Each

ver-

ti-

cal

junc-

tion

is

bi-

641

ased

by

a

cur-

645

rent

$I_b <$

I_c

to

sup-

press

its

plasma

fre-

quency.

The

ge-

o-

met-

659

ric

in-

661

duc-

tance

is

very

665

small

and

ne-

glected.

The

669

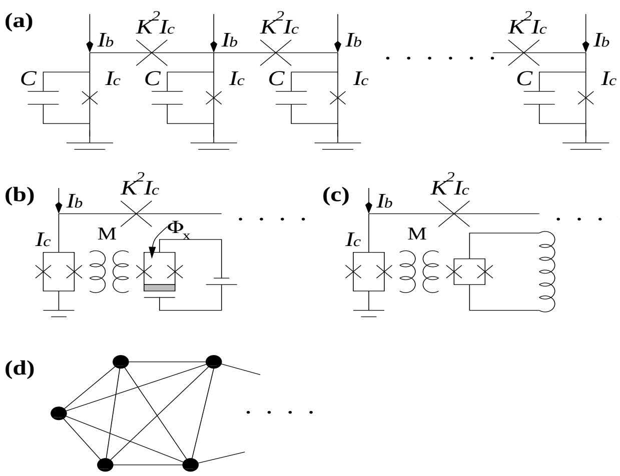

Fig. 1. (a) A simple Josephson junction array consisting of N vertical and $N-1$ horizontal junc-

po-

ten-

tial

673

en-

ergy

of

the

Joseph-

son

junc-

tion

ar-

ray

in

Fig.

1

is

a

sum

of

the

Joseph-

son

tun-

nel-

ing

en-

er-

gies

699

of

the

junc-

tions,

given

by

$$V = -E_J \sum_{i=0}^{N-1} (i_b \theta_i + \cos\theta_i) - K^2 E_J \sum_{i=0}^{N-2} \cos(\theta_i - \theta_{i+1}), (1)$$

where

$$E_J =$$

$I_c\Phi_0/2\pi$

and

$i_b =$

I_b/I_c.

The

equi-

lib-

rium

val-

ues

for

719

the

junc-

tion

phases

are

de-

725

ter-

mined

727

by

$\partial V / \partial \theta_i =$

729

0,

which

731

is

just

733

the

cur-

rent

con-

ser-

va-

tion

con-

741

di-

tion

at

each

node:

i_b-

$\sin\theta_i -$

$\sin(\theta_i - \theta_{i-1}) -$

$$\sin\left(\theta_i - \theta_{i+1}\right) =$$

0.

(When

$i =$

753

0

or

$N-$

1

there

is

only

one

761

hor-

i-

zon-

tal

cur-

rent.)

In

our

setup,

the

equi-

lib-

rium

val-

ues

for

the

phases

are

$\theta_i^{(0)} =$

$\theta^{(0)} =$

$\arcsin i_b$

for

$i =$

$0, 1, ...N-$

1.

If

we

789

are

in-

791

ter-

ested

in

the

795

small

os-

cil-

la-

799

tions

of

the

phases,

we

fol-

805

low

the

stan-

dard

pro-

ce-

dure

ex-

pand-

ing

815

V

to

sec-

ond

or-

der,

$V =$

$\frac{1}{2}\sum_{i,j=0}^{N-1} V_{ij}\varphi_i\varphi_j,$

where

$\varphi_i =$

θ_i-

$\theta_i^{(0)}$

is

the

829

small

dis-

831

place-

ment

833

of

the

phase

from

its

equi-

839

lib-

rium

841

value

and

$$\frac{(V)_{ij}}{E_J K^2} = \delta_{ij}(2 + \cos\theta^{(0)}/K^2 - \delta_{i,0} - \delta_{i,N-1}) - \delta_{i,j-1} - \delta_{i,j+1}. (2)$$

Notice

that

the

hor-

i-

zon-

tal

851

junc-

tions

are

much

855

larger

than

the

ver-

ti-

cal

ones.

In

863

the

“wash

865

board”

anal-

867

ogy

this

cor-

re-

871

sponds

to

a

large

875

“mass”

for

the

hor-

i-

zon-

881

tal

junc-

tions.

There-

fore

in

887

cal-

cu-

889

lat-

890

ing

891

the

ki-

893

netic

en-

895

ergy

of

897

the

sys-

899

tem

we

901

need

only

con-

sider

905

the

ver-

907

ti-

cal

909

junc-

tions

911

(as

ver-

i-

fied

915

by

nu-

mer-

i-

919

cal

cal-

921

cu-

la-

tions):

$T =$

$(1/2)C(\Phi_0/2\pi)^2\sum_{i=0}^{N-1}\dot{\varphi}_i^2$.

From

927

the

po-

ten-

930

tial

and

ki-

933

netic

en-

935

er-

gies

937

we

can

939

solve

for

941

the

nor-

943

mal

modes

945

of

the

947

sys-

tem,

949

whose

spec-

trum

is

given

954

by

$\nu_s =$

$$\omega_p\sqrt{1 + (4K^2/\cos\theta^{(0)})\sin^2(s\pi/2N)},$$

where

the

plasma

fre-

quency

$\omega_p =$

$\sqrt{2\pi I_c/\Phi_0 C}(1-$

$i_b^2)^{1/4}$

965

and

$s =$

$0, 1, ..., N-$

1.

Let

970

the

or-

thonor-

973

mal-

974

ized

nor-

mal

977

mode

eigen-

vec-

tors

be

de-

983

noted

b_i^s.

The

low-

est

mode,

989

whose

fre-

quency

is

ω_p,

cor-

995

re-

sponds

997

to

the

cen-

ter

1001

of

mass

1003

mo-

tion

1005

of

the

phases:

$b^0 =$

$(1, 1, ..., 1)/\sqrt{N}$.

The

quan-

tum

me-

chan-

i-

cal

1017

prop-

er-

1019

ties

of

1021

the

small

1023

col-

lec-

1025

tive

vi-

1027

bra-

tional

1029

modes

1030

of

1031

the

phases

1033

can

now

be

eval-

u-

ated

1039

by

in-

tro-

duc-

ing

the

op-

er-

1047

a-

tor

$\hat{\varphi}_i =$

$(2\pi/\Phi_0)\sum_{s=0}^{N-1}\sqrt{\hbar/2C\nu_s}b_i^s(a_s+$

$a_s^\dagger)$,

where

a_s

is

1055

the

an-

ni-

hi-

la-

tion

op-

er-

a-

tor

for

1066

the

1067

sth

mode.

We

pro-

1071

pose

to

1073

use

the

cen-

ter

1077

of

1078

mass

1079

mo-

tion

1081

mode

of

1083

the

Joseph-

1085

son

junc-

tion

ar-

1089

ray

to

1091

cou-

ple

su-

per-

1095

con-

duct-

1097

ing

qubits,

in

1100

anal-

1101

ogy

1102

to

1103

ion

1104

trap

1105

quan-

tum

1107

com-

puter

1109

[23].

Since

1111

the

1112

cen-

1113

ter

of

1115

mass

mo-

1117

tion

mode

1119

is

an

1121

equal

weight

su-

per-

1125

po-

si-

1127

tion

1128

of

1129

the

junc-

1131

tion

phases,

its

1134

qual-

1135

ity

fac-

1137

tor

is

as

high

as

that

1143

of

the

1145

in-

di-

1147

vid-

ual

1149

junc-

tions.

1151

There-

fore,

1153

this

ap-

1155

proach

al-

1157

lows

us

1159

to

take

1161

ad-

van-

1163

tage

of

1165

the

high

1167

qual-

ity

1169

Joseph-

1170

son

1171

junc-

tions

re-

quired

for

the

1177

su-

per-

1179

con-

duct-

ing

qubits

1183

to

re-

1185

al-

ize

1187

a

1188

high

1189

qual-

1190

ity

res-

onator.

1193

Note

in

1195

the

1196

above

we

have

as-

sumed

1201

that

1202

all

Joseph-

son

1205

junc-

tions

1207

in

1208

the

1209

ar-

ray

1211

are

iden-

ti-

cal.

1215

In

re-

1217

al-

ity,

1219

this

will

1221

not

be

1223

the

1224

case

1225

due

to

un-

1228

avoid-

1229

able

fab-

ri-

ca-

1233

tion

er-

rors.

How-

1237

ever

the

1239

crit-

i-

cal

cur-

1243

rents

of

1245

the

junc-

1247

tions

can

1249

be

1250

tuned

1251

by

a

1253

mag-

netic

1255

field

so

1257

that

the

ef-

fec-

1261

tive

1262

Joseph-

1263

son

en-

er-

gies

1267

of

the

junc-

tions

1271

can

be

equal-

ized.

1275

The

ef-

1277

fect

of

1279

any

resid-

1281

ual

asym-

1283

me-

try

1285

can

be

1287

es-

ti-

1289

mated

1290

by

1291

per-

tur-

1293

ba-

tion

1295

the-

ory.

1297

As

long

1299

as

1300

the

1301

am-

pli-

1303

tude

of

1305

the

tran-

si-

tion

1309

ma-

1310

trix

1311

el-

e-

1313

ment

due

1315

to

the

asym-

me-

1319

try

is

1321

much

smaller

1323

than

the

1325

en-

ergy

1327

gap

be-

1329

tween

1330

the

1331

cen-

ter

1333

of

1334

mass

mo-

tion

1337

mode

and

1339

higher

modes,

1341

the

en-

1343

ergy

and

1345

wave-

func-

1347

tion

of

1349

the

1350

cen-

1351

ter

of

1353

mass

mo-

1355

tion

1356

mode

1357

re-

main

1359

close

1360

to

1361

un-

per-

turbed.

In

1365

or-

der

1367

to

im-

ple-

1370

ment

pro-

to-

1373

cols

de-

1375

vel-

oped

1377

in

su-

1379

per-

1380

con-

duct-

ing

1383

qubit

1384

based

1385

cavity-

1386

QED

1387

[13,14,15,16],

we

1389

need

to

1391

cou-

ple

1393

the

su-

1395

per-

con-

1397

duct-

ing

1399

qubits

1400

to

1401

the

cen-

1403

ter

1404

of

1405

mass

mo-

1407

tion

1408

mode

1409

of

1410

the

1411

junc-

tion

1413

ar-

ray

1415

in

a

1417

way

1418

such

1419

that

1420

they

1421

can

ex-

1423

change

en-

1425

ergy.

This

1427

is

shown

1429

in

1430

Fig.

1431

1

1432

(b)

and

(c)

1435

for

both

1437

charge

and

1439

flux

qubits.

1441

Here

each

ver-

ti-

1445

cal

1446

junc-

1447

tion

in

1449

the

1450

junc-

1451

tion

ar-

1453

ray

is

re-

placed

1457

by

a

1459

small

self-

1461

inductance

1462

dc-

SQUID

and

cou-

pled

1467

in-

1468

duc-

1469

tively

to

1471

a

su-

1473

per-

con-

1475

duct-

ing

qubit.

Con-

1479

sider

1480

the

1481

charge

1482

qubit

1483

whose

1484

Hamil-

1485

to-

1486

nian

1487

is

$H_Q =$

1489

$-B^z\sigma^z/2-$

$B^x \sigma^x/2,$

where

B^z

1493

and

B^x

1495

are

de-

1497

ter-

mined

1499

by

1500

the

1501

gate

1502

volt-

age

1504

and

1505

Joseph-

1506

son

1507

en-

ergy

1509

of

1510

the

1511

charge

qubit.

1513

When

its

1515

en-

1516

ergy

1517

B^z

is

1519

tuned

1520

close

1521

to

$\nu_0 =$

1523

ω_p

and

1525

its

1526

dc-

1527

SQUID

is

1529

bi-

ased

1531

at

1532

$\Phi_0/2$

1533

(in-

clud-

1535

ing

1536

the

1537

flux

due

1539

to

1540

the

1541

junc-

tion

1543

ar-

1544

ray’s

1545

bias

cur-

1547

rent

I_b),

1549

the

1550

in-

1551

duc-

tive

1553

cou-

pling

1555

re-

1556

sults

1557

in

1558

a

1559

cou-

1560

pling

1561

Hamil-

to-

1563

nian

$H_c =$

$-g(a\sigma^{+}+$

$a^\dagger\sigma^-)$,

where

1568

$g =$

$(M/2)(I_c \cos\theta^{(0)})I_c^Q(2\pi/\Phi_0)\sqrt{\hbar/2C\omega_p N},$

M

is

the

1573

mu-

1574

tual

1575

in-

1576

duc-

tance,

I_c^Q

is

1580

the

1581

crit-

i-

cal

cur-

1585

rent

of

the

1588

dc-

1589

SQUID

1590

junc-

1591

tions

1592

of

1593

the

1594

charge

1595

qubit,

and

1597

a

is

1599

the

an-

1601

ni-

hi-

1603

la-

tion

1605

op-

er-

1607

a-

tor

1609

for

1610

the

1611

cen-

ter

1613

of

1614

mass

1615

mo-

1616

tion

1617

mode

of

1619

the

junc-

tion

ar-

1623

ray.

In

de-

riv-

1627

ing

1628

H_c,

1629

we

have

1631

used

the

ro-

1634

tat-

ing

1636

wave

1637

ap-

prox-

1639

i-

ma-

1641

tion

1642

to

1643

drop

terms

1645

that

os-

1647

cil-

1648

late

1649

at

1650

high

1651

fre-

1652

quen-

cies.

1654

For

1655

the

1656

flux

1657

qubit

1658

case

1659

shown

in

1661

Fig.

1

1663

(b),

we

1665

can

de-

rive

1668

the

same

cou-

pling

1672

Hamil-

1673

to-

1674

nian,

1675

with

a

1677

cou-

pling

1679

co-

ef-

1681

fi-

1682

cient

1683

which

is

1685

pro-

1686

por-

1687

tional

1688

to

the

1690

mu-

1691

tual

1692

in-

1693

duc-

1694

tance

1695

and

can

1697

be

1698

eval-

u-

ated

in

terms

of

1704

the

1705

qubit

pa-

1707

ram-

1708

e-

1709

ters

[24].

With

the

1713

above

de-

1715

sign

we

1717

then

1718

have

1719

a

struc-

ture

in

1723

close

anal-

1725

ogy

1726

to

1727

the

ion

1729

trap

1730

quan-

tum

com-

puter

1734

in

1735

which

1736

the

1737

qubits

com-

1739

mu-

ni-

1741

cate

1742

through

the

cen-

1745

ter

1746

of

mass

1748

phonon

mode.

To

re-

1752

al-

1753

ize

1754

a

1755

uni-

ver-

sal

quan-

tum

com-

1761

puter,

we

1763

can

use

ei-

1766

ther

1767

the

res-

o-

nant

1771

[13,14]

or

dis-

per-

1775

sive

1776

[13,15,16]

1777

in-

ter-

1779

ac-

tion

1781

be-

tween

1783

the

1784

qubits

and

1786

the

1787

junc-

tion

1789

ar-

1790

ray

mode.

The

1793

in-

1794

ter-

ac-

tion

is

switched

1799

on

and

off

1802

by

1803

tun-

ing

the

en-

1807

ergy

1808

of

1809

the

qubit

1811

into

1812

and

1813

out

1814

of

res-

o-

1817

nance

1818

with

1819

the

1820

junc-

1821

tion

ar-

ray.

1824

Typical

1825

val-

ues

1827

for

ω_p

and

1830

qubit

1831

en-

er-

1833

gies

1834

can

be

1836

cho-

sen

1838

to

1839

be

1840

up

1841

to

1842

10GHz.

1843

The

1844

cou-

1845

pling

1846

strength

1847

g

can

1849

be

1850

tens

1851

of

1852

mega-

1853

hertz

1854

[15,25].

When

1856

the

sys-

1858

tem

1859

scales

1860

up

1861

$(N$

1862

in-

1863

creases),

1864

the

1865

en-

1866

ergy

1867

gap

1868

be-

tween

1870

the

1871

cen-

1872

ter

1873

of

1874

mass

1875

mo-

tion

1877

mode

1878

and

the

1880

higher

1881

modes

should

1883

re-

1884

main

1885

much

1886

greater

1887

than

1888

the

cou-

pling

1891

strength

1892

g

1893

to

1894

avoid

1895

ex-

1896

ci-

1897

ta-

1898

tion

1899

of

up-

1901

per

modes.

When

1904

N

is

1906

large,

the

1908

en-

1909

ergy

1910

dif-

1911

fer-

1912

ence

1913

be-

1914

tween

1915

the

1916

low-

1917

est

1918

two

1919

modes

is

1921

$\Delta\nu_{01} =$

$(\pi^2 K^2/2N^2 \cos\theta^{(0)})\omega_p.$

For

$K =$

1925

20

[26]

and

$i_b =$

1929

0.97,

1930

this

1931

im-

1932

plies

1933

an

1934

up-

1935

per

1936

limit

1937

of

1938

a

1939

few

hun-

1941

dred

1942

for

1943

$N.$

1944

To

1945

re-

1946

lax

this

limit,

1949

we

can

1951

use

1952

more

1953

com-

1954

pli-

1955

cated

1956

de-

1957

signs

1958

than

1959

the

1960

sim-

1961

ple

1962

1d

1963

ar-

1964

ray

1965

in

1966

Fig.

1967

1

1968

(a).

For

1970

in-

1971

stance,

1972

we

1973

can

1974

con-

sider

1976

a

1977

net-

1978

work

1979

of

N

1981

su-

1982

per-

1983

con-

1984

duct-

1985

ing

1986

is-

1987

lands

1988

in

1989

which

1990

each

1991

pair

1992

of

1993

nodes

1994

is

1995

cou-

1996

pled

1997

by

1998

a

1999

Joseph-

2000

son

2001

junc-

2002

tion,

2003

as

2004

shown

2005

in

2006

Fig.

2007

1

2008

(d).

Each

2010

is-

2011

land

2012

is

2013

still

2014

grounded

2015

though

2016

a

2017

junc-

2018

tion

2019

whose

2020

plasma

2021

fre-

2022

quency

2023

is

2024

$\omega_p,$

2025

and

2026

the

2027

Joseph-

2028

son

2029

en-

2030

ergy

2031

of

the

cou-

pling

2035

junc-

tions

is

2038

K^2

that

2040

of

2041

the

ground-

2043

ing

2044

junc-

tions.

In

this

case,

2049

the

2050

cen-

2051

ter

2052

of

2053

mass

mo-

tion

mode

2057

re-

2058

mains

at

ω_p

and

all

2063

higher

2064

modes

2065

are

pushed

2067

up

to

2069

a

fre-

quency

$\omega_p\sqrt{1+NK^2/\cos\theta^{(0)}}$.

The

num-

ber

2076

of

2077

junc-

2078

tions

2079

re-

quired

2081

is

2082

on

2083

the

or-

2085

der

2086

$N^2/2.$

2087

In

2088

the

2089

above,

we

take

ad-

van-

2094

tage

2095

of

the

2097

macro-

scopic

2099

quan-

tum

be-

2102

hav-

2103

ior

2104

of

2105

the

2106

Joseph-

2107

son

2108

junc-

2109

tion

2110

phases

2111

to

con-

2113

struct

an

2115

anal-

ogy

2117

of

the

2119

ion

2120

trap

2121

quan-

tum

2123

com-

puter.

2125

We

can

2127

push

this

2129

con-

cept

2131

fur-

ther

and

con-

2135

sider

us-

ing

Joseph-

2139

son

junc-

2141

tion

ar-

2143

rays

to

2145

sim-

u-

2147

late

2148

the

2149

dy-

nam-

2151

ics

of

2153

phys-

i-

2155

cal

sys-

tems.

2158

Con-

2159

sider

a

2161

1d

ar-

2163

ray

of

qubits.

In

2167

the

spin

2169

1/2

rep-

re-

sen-

2173

ta-

tion,

2175

the

dy-

2177

nam-

ics

of

the

2181

qubits

are

2183

de-

scribed

2185

by

the

2187

spin

op-

2189

er-

a-

tors

S_i^x,

2193

S_i^y

and

2195

S_i^z

whose

2197

com-

mu-

2199

ta-

tion

re-

la-

tions

are

2205

de-

fined

2207

by

$$[S_i^\alpha, S_j^\beta] =$$

$(i/2)\delta_{ij}\epsilon_{\alpha\beta\gamma}S_i^{\gamma}$

for

2211

$\alpha, \beta, \gamma =$

x, y, z

and

$i, j =$

2215

$0, 1, ..., N-$

1.

2217

By

us-

2219

ing

2220

a

Jordan-

2222

Wigner

2223

trans-

for-

2225

ma-

tion

2227

[27],

we

2229

can

2230

map

2231

this

qubit

2233

ar-

ray

2235

to

a

2237

col-

lec-

2239

tion

2241

spin-

less

2243

fermions

an-

2245

ni-

hi-

2247

lated

2248

by

2249

the

op-

er-

a-

2253

tors

$$f_n = S_n^- K(n), (3)$$

where

$S_n^{\pm} =$

2257

$S_n^x \pm$

iS_n^y

and

2260

$K(n)$

2261

is

a

2263

non-

lo-

2265

cal

op-

2267

er-

a-

2269

tor

2270

de-

2271

fined

by

$K(n) =$

$exp[i\pi \sum_{m=0}^{n-1} f_m^+ f_m] =$

$exp[i\pi \sum_{m=0}^{n-1}(S_m^z +$

1/2)].

It

can

2279

be

ver-

2281

i-

fied

2283

that

f_n's

sat-

2286

isfy

the

anti-

commutation

re-

la-

tions

2293

re-

quired

for

fermion

2297

op-

er-

2299

a-

tors.

The

2302

fermion

num-

2304

ber

2305

op-

er-

2307

a-

tor

is

sim-

2311

ply

$f_n^\dagger f_n =$

2313

S_n^z+

1/2.

Now,

if

we

cou-

2319

ple

the

2321

qubits

2322

by

2323

near-

est

2325

neigh-

bor

2327

XXZ

in-

2329

ter-

ac-

2331

tions,

the

2333

Hamil-

to-

2335

nian

2336

of

2337

the

sys-

tem

is

2341

$H =$

$-J_{xy} \sum_i (S_i^x S_{i+1}^x +$

2343

$S_i^y S_{i+1}^y)+$

$J_z \sum_i S_i^z S_{i+1}^z$.

2345

Un-

der

2347

the

2348

Jordan-

2349

Wigner

2350

trans-

2351

for-

ma-

2353

tion,

the

2355

sys-

tem

2357

Hamil-

to-

2359

nian

is

2361

trans-

formed

2363

to

$$H = -(J_{xy}/2)\sum_n (f_n^\dagger f_{n+1} + f_{n+1}^\dagger f_n) + J_z \sum_n (f_n^\dagger f_n - 1/2)(f_{n+1}^\dagger f_{n+1} - 1/2).(4)$$

As

can

be

seen

2369

the

cou-

2371

pling

in

the

XY

2375

di-

rec-

2377

tions

trans-

2379

forms

into

2381

hop-

ing

2383

of

the

fermions

be-

tween

neigh-

2389

bor-

ing

2391

sites

and

2393

the

2394

Ising

cou-

pling

2397

causes

near-

2399

est

neigh-

bor

in-

2403

ter-

ac-

2405

tions

be-

2407

tween

the

fermions.

An

2410

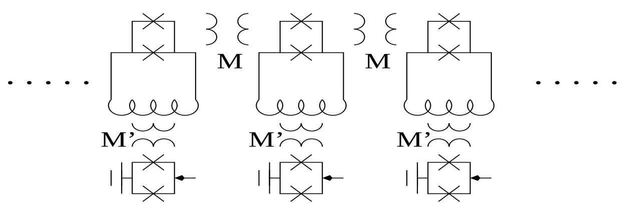

Fig. 2. A 1d rf-SQUID qubit array inductively coupled to local Josephson junction oscillators. The dc-SQUIDs of the qubits are inductively cou-

in-

2412

ter-

est-

ing

2415

sce-

2416

nario

arises

when

we

cou-

2421

ple

a

2423

su-

per-

con-

duct-

ing

2428

qubit

2429

ar-

ray

and

“phonon

2433

modes”

re-

al-

ized

2437

by

2438

small

os-

cil-

2441

la-

tions

2443

of

Joseph-

2445

son

junc-

2447

tion

phases,

as

shown

2451

in

Fig.

2453

2.

Since

2455

the

2456

qubit

2457

ar-

2458

ray

2459

can

be

2461

trans-

formed

into

a

2465

fermion

sys-

tem,

2468

this

2469

al-

lows

2471

us

to

2473

sim-

u-

2475

late

a

2477

fam-

ily

2479

of

2480

solid

2481

state

2482

prob-

2483

lems

con-

2485

cern-

ing

2487

spin-

2488

less

2489

electron-

2490

phonon

and

electron-

2493

electron

in-

2495

ter-

ac-

tions.

The

2499

macro-

2500

scopic

na-

ture

2503

of

the

2505

sys-

tem

2507

and

2508

the

fine

2510

con-

trol

and

2513

mea-

sure-

2515

ment

tech-

2517

niques

de-

vel-

2520

oped

2521

in

su-

per-

con-

duct-

ing

quan-

tum

2529

in-

for-

ma-

tion

2533

pro-

cess-

ing

al-

low

us

to

ex-

2541

am-

ine

2543

the

fermion

2545

dy-

nam-

2547

ics

di-

rectly

and

2551

closely

to

ob-

tain

2555

vi-

tal

in-

for-

2559

ma-

tion

2561

about

the

2563

sys-

tem.

For

this

2567

pur-

pose,

2569

we

need

2571

to

be

2573

care-

ful

2575

with

a

2577

few

is-

sues.

The

2581

first

is

that

in

2585

a

phys-

i-

cal

2589

sys-

tem

the

2592

elec-

2593

tron

num-

2595

ber

is

2597

con-

served;

2599

how-

ever

2601

a

$B^x \sigma^x$

2603

term

of

2605

the

2606

qubit

can

flip

2609

its

ba-

2611

sis

states

de-

fined

2615

by

the

2617

eigen-

states

2619

of

σ^z.

2621

Ac-

cord-

ing

to

the

2626

Jordan-

2627

Wigner

trans-

for-

ma-

2631

tion

Eq.

2633

(3)

the

2635

flip-

ping

2637

of

the

2639

qubit

state

2641

cor-

re-

2643

sponds

to

2645

the

cre-

2647

ation

or

2649

an-

ni-

2651

hi-

la-

2653

tion

of

2655

a

fermion.

There-

fore,

2659

we

2660

must

2661

keep

B^x

2663

of

the

2665

qubits

zero.

This

is

re-

al-

2671

ized

2672

by

2673

us-

ing

2675

small

self-

inductance

dc-

2679

SQUIDs

for

2681

the

2682

qubits

and

bi-

2685

as-

ing

them

ap-

pro-

pri-

2691

ately

so

that

the

2695

B^x

field

2697

of

the

2699

qubit

2700

is

2701

much

smaller

2703

[15]

2704

than

2705

other

en-

ergy

scales

of

the

2711

sys-

tem

and

thus

2715

can

be

2717

ne-

glected

2719

for

the

2721

time

2722

scale

2723

of

in-

2725

ter-

est.

To

fur-

ther

sup-

2731

press

fermion

2733

cre-

ation

and

an-

2737

ni-

hi-

la-

tion,

2741

we

may

2743

choose

B^z

2745

to

be

2747

large,

which

2749

makes

2750

changes

2751

in

the

to-

tal

2755

num-

2756

ber

2757

of

2758

fermions

2759

en-

er-

2761

get-

i-

2763

cally

for-

2765

bid-

den.

An-

2768

other

2769

point

is

2771

that

the

2773

cou-

pling

2775

be-

tween

2777

the

qubit

and

the

2781

Joseph-

son

2783

phase

should

2785

not

al-

2787

low

them

to

ex-

2791

change

ex-

ci-

ta-

tions,

since

2797

in

a

2799

phys-

i-

2801

cal

sys-

2803

tem

2804

no

2805

pro-

cess

2807

hap-

2808

pens

in

which

2811

an

2812

elec-

2813

tron

is

cre-

ated

2817

(an-

ni-

2819

hi-

lated)

and

a

2823

phonon

is

an-

ni-

hi-

lated

(cre-

ated).

2831

In

the

2833

fol-

low-

ing

we

2837

fo-

cus

2839

on

2840

the

2841

1d

Hol-

2843

stein

model

2845

of

spin-

2847

less

2848

elec-

trons,

2850

a

2851

prob-

2852

lem

2853

of

both

the-

o-

2857

ret-

i-

2859

cal

and

2861

prac-

ti-

2863

cal

in-

2865

ter-

est

[30,31].

The

2869

Hamil-

to-

2871

nian

of

2873

the

sys-

2875

tem

is

$$H = -t\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + \omega \sum_i a_i^\dagger a_i - g\sum_i (c_i^\dagger c_i - 1/2)(a_i + a_i^\dagger), (5)$$

where

2879

c_i

de-

2881

stroys

a

2883

fermion

at

2885

site

2886

i

and

a_i

de-

stroys

a

lo-

2893

cal

2894

phonon

2895

of

fre-

2897

quency

$\omega.$

This

sys-

2901

tem

at

2903

half

fill-

ing

has

2907

two

phases

2909

de-

2910

pend-

ing

on

2913

the

ra-

2915

tio

g/ω.

When

2918

$g <$

2919

g_c,

the

ground

2922

state

2923

has

a

2925

metal-

lic

(Lut-

tinger

2929

liq-

2930

uid)

phase.

When

2933

g

ex-

2935

ceeds

g_c,

it

ex-

2939

hibits

an

in-

su-

2943

lat-

ing

2945

phase

with

2947

charge

2948

den-

2949

sity

2950

wave

2951

(CDW)

long

2953

range

or-

der

[30,31].

Though

it

2959

can

be

2961

solved

an-

a-

2964

lyt-

i-

cally

2967

in

cer-

tain

lim-

2971

its,

for

2973

generic

pa-

2975

ram-

e-

ters

the

2979

ac-

cu-

rate

2982

de-

ter-

mi-

na-

tion

2987

of

the

2989

crit-

i-

cal

2992

cou-

2993

pling

strength

and

en-

2997

ergy

2998

gaps

is

dif-

fi-

cult

and

only

3005

lim-

3006

ited

3007

size

sys-

3009

tems

at

3011

half

3012

fill-

3013

ing

have

3015

been

stud-

ied

nu-

3019

mer-

i-

3021

cally

[30,31].

We

can

3025

con-

struct

an

ar-

ti-

fi-

3031

cial

re-

al-

iza-

tion

of

3037

the

3038

1d

3039

Hol-

3040

stein

3041

model

us-

ing

a

3045

su-

per-

con-

duct-

3049

ing

3050

qubit

3051

ar-

3052

ray

and

lo-

3055

cal

os-

cil-

la-

3059

tor

modes

re-

al-

3063

ized

3064

by

Joseph-

son

3067

junc-

tions,

as

shown

3071

in

Fig.

2.

For

3075

clar-

ity,

3077

rf-

SQUID

3079

qubits

are

shown,

whose

3083

dc-

SQUIDs

are

cou-

3087

pled

3088

by

3089

a

tun-

3091

able

mu-

3093

tual

in-

duc-

tance.

The

tun-

3099

able

mu-

3101

tual

in-

3103

duc-

tances

3105

can

be

re-

al-

3109

ized

3110

by

us-

ing

3113

a

flux

trans-

3116

former

3117

in-

ter-

3119

rupted

by

3121

a

dc-

3123

SQUID

whose

3125

crit-

i-

3127

cal

cur-

rent

can

be

3132

tuned

3133

by

a

3135

flux

3136

bias,

as

3138

dis-

cussed

in

3141

[24,28,29].

As

a

re-

3145

sult

of

3147

this

inter-

3149

qubit

3150

cou-

3151

pling

and

3153

the

3154

qubit

3155

bias,

the

3157

qubit

ar-

3159

ray

Hamil-

to-

nian

3163

is

$H_Q =$

$-\sum_i B^z S_i^z-$

$J \sum_i S_i^x S_{i+1}^x,$

where

B^z

3169

is

3170

de-

ter-

mined

3173

by

the

3175

flux

bias

3177

of

the

3179

qubits

and

3181

J

is

3183

pro-

por-

3185

tional

to

the

tun-

3189

able

3190

mu-

3191

tual

in-

3193

duc-

tance

3195

M.

The

3197

lo-

cal

3199

phonon

3200

modes

are

re-

3203

al-

3204

ized

3205

by

Joseph-

3207

son

junc-

tions

whose

os-

cil-

3213

la-

tion

fre-

quency

is

3218

the

plasma

fre-

quency

ω_p.

These

junc-

tions

should

3227

have

a

3229

deep

po-

3231

ten-

tial

well

in

or-

der

3237

to

guar-

an-

3240

tee

3241

that

their

3243

be-

hav-

3245

ior

is

3247

close

to

3249

har-

3250

monic

even

when

3253

the

ex-

3255

ci-

ta-

3257

tion

num-

3259

ber

is

3261

not

small.

Al-

ter-

na-

tively,

if

we

are

also

in-

ter-

3273

ested

in

3275

study-

ing

the

ef-

fect

of

3281

non-

har-

monic-

ity

3285

of

the

3287

phonon

modes

on

the

sys-

tem

3293

be-

hav-

3295

ior

3296

[32],

3297

we

3298

may

3299

use

an

3301

in-

duc-

3303

tor

(L)

3305

shunted

3306

dc-

3307

SQUID.

In

3309

this

case,

3311

un-

der

the

con-

3315

di-

tion

3317

$LI_c \ll$

3318

$\Phi_0/2\pi,$

3319

the

Hamil-

3321

to-

3322

nian

3323

for

the

3325

phonon

mode

3327

is

$Q^2/2C_J+$

$\Phi^2/2L'-$

$(I_c/24)(2\pi/\Phi_0)^3\Phi^4,$

where

Q

and

Φ

3335

are

the

charge

and

3339

flux

across

3341

the

dc-

3343

SQUID,

C_J

is

the

3347

to-

tal

3349

junc-

tion

3351

ca-

pac-

3353

i-

tance

and

the

3357

ef-

fec-

3359

tive

in-

3361

duc-

tance

$L' =$

$L(1-$

$2\pi L I_c/\Phi_0$).

The

3367

an-

har-

3369

monic-

ity

3371

can

be

ad-

justed

3375

by

tun-

3377

ing

the

crit-

i-

3381

cal

cur-

3383

rent

I_c

3385

of

the

3387

dc-

SQUID.

The

lo-

3391

cal

Joseph-

3393

son

junc-

3395

tion

os-

cil-

la-

3399

tors

are

3401

cou-

pled

to

the

3405

main

loops

3407

of

the

3409

rf-

3410

SQUID

3411

qubits

us-

3413

ing

a

tun-

able

3417

mu-

3418

tual

in-

duc-

tance

as

3423

shown

3424

in

3425

Fig.

2.

This

in-

3429

tro-

duces

a

cou-

pling

Hamil-

to-

nian

pro-

por-

tional

to

3441

$S^z\Phi,$

with

3443

a

cou-

pling

co-

3447

ef-

fi-

3449

cient

pro-

3451

por-

tional

3453

to

3454

the

3455

mu-

3456

tual

in-

3458

duc-

3459

tance

$M'[24]$.

Af-

3462

ter

3463

quan-

tiz-

ing

the

lo-

cal

3469

phonon

modes

and

ap-

3473

ply-

ing

3475

the

3476

Jordan-

3477

Wigner

trans-

3479

for-

ma-

tion,

we

ob-

tain

3485

the

Hamil-

to-

nian

3489

of

3490

the

3491

1d

Hol-

3493

stein

3494

model

3495

Eq.

(5),

with

the

3499

hop-

ping

3501

strength

$t =$

3503

$J/4$

and

3505

the

3506

phonon

fre-

quency

3509

$\omega =$

ω_p.

Since

3512

the

qubit-

3514

qubit

and

3516

qubit-

3517

phonon

in-

3519

ter-

ac-

tions

are

re-

al-

3525

ized

with

3527

tun-

able

3529

cou-

plings,

the

ra-

3533

tio

of

t

and

3537

g

to

3539

ω

in

3541

the

Hol-

3543

stein

model

can

be

3547

tuned

in

3549

a

3550

wide

3551

range.

This

3553

al-

lows

3555

us

to

3557

ex-

plore

a

large

phase

space

3563

of

the

3565

sys-

tem.

To

sim-

u-

late

3571

the

3572

1d

3573

Hol-

3574

stein

3575

model,

we

first

set

3579

the

num-

ber

of

3583

spin-

less

3585

fermions

in

3587

the

sys-

tem.

This

is

3592

done

3593

by

ini-

3595

tial-

iz-

ing

3598

half

the

3600

qubits

in

the

$S^z =$

3604

1/2

3605

state

and

the

3608

other

3609

half

3610

in

3611

the

$S^z =$

3613

$-1/2$

state

3615

(as-

sum-

ing

we

are

in-

3621

ter-

ested

3623

in

the

half

fill-

ing

case),

3629

which

can

3631

be

ac-

3633

com-

plished

3635

by

ap-

3637

ply-

ing

3639

strong

B^z

fields

3642

to

the

qubits

or

3646

by

per-

form-

3649

ing

mea-

sure-

ments.

Then

t

and

g

are

3658

turned

3659

on

(by

3661

tun-

ing

3663

the

inter-

3665

qubit

and

3667

qubit

phonon

3669

cou-

plings)

slowly

un-

3673

til

the

in-

tended

3677

point

(t, g, ω)

3679

in

the

3681

phase

space

is

reached.

The

phase

of

the

sys-

tem

can

be

in-

ferred

3695

by

mea-

3697

sur-

ing

the

fermion

3701

num-

bers

$f_n^\dagger f_n,$

or

3705

$S_n^z,$

which

3707

are

in-

de-

pen-

3711

dent

of

3713

n

3714

(1/2

for

half

3717

fill-

ing)

for

the

metal-

lic

3723

phase

and

vary-

ing

with

n

for

the

in-

su-

3733

lat-

ing

phase.

The

ex-

ci-

ta-

tion

en-

ergy

3743

of

the

3745

sys-

tem

can

be

3749

de-

ter-

mined

by

3753

spec-

tro-

3755

scopic

3756

meth-

3757

ods

like

in

[18].

In

con-

3763

clu-

sion,

3765

we

have

3767

shown

that

3769

prop-

erly

en-

gi-

neered

Joseph-

son

junc-

3777

tion

ar-

3779

rays

can

be

used

3783

for

quan-

3785

tum

ma-

3787

nip-

u-

la-

tion

and

sim-

u-

la-

tion.

The

dis-

cussed

pro-

3800

to-

3801

cols

re-

quire

only

lim-

ited

3807

con-

trol

of

the

sys-

tem

and

3814

pro-

3815

vide

in-

ter-

est-

3819

ing

op-

por-

tu-

ni-

ties

for

su-

per-

con-

3829

duct-

ing

3831

quan-

tum

3833

in-

for-

ma-

tion

pro-

cess-

ing.

The

3841

au-

thors

3843

ac-

knowl-

3845

edge

fi-

nan-

3848

cial

3849

sup-

port

3851

from

the

3853

Packard

Foun-

da-

tion.

References

[1]
Y.

Makhlin,

G.

Schön,

and

A.

Shnirman,

Rev.

Mod.

Phys.

73,

357

(2001).

[2]

Y.

Nakamura,

Y.

A.

Pashkin,

and

J.

S.

Tsai,

Nature

(London)

398,

786

(1999).

[3]

J.

R.

Friedman,

V.

Patel,

W.

Chen,

S.

K.

Tolpygo,

and

J.

E.

Lukens,

Nature

406,

43

(2000).

[4]

C.

H.

van

der

Wal,

A.

C.

J.

ter

Haar,

F.

K.

Wilhelm,

R.

N.

Schouten,

C.

J.

P.

M.

Harmans,

T.

P.

Orlando,

S.

Lloyd,

and

J.

E.

Mooij,

Science

290,

773

(2000).

[5]

D.

Vion,

A.

Aassime,

A.

Cottet,

P.

Joyez,

H.

Pothier,

C.

Urbina,

D.

Esteve,

and

M.

H.

Devoret,

Science

296,

886

(2002).

[6]

Y.

Yu,

S.

Han,

X.

Chu,

S-

I.Chu,

and

Z.

Wang,

Science

296,

889

(2002).

[7]

J.

M.

Martinis

S.

Nam,

J.

Aumentado,

and

C.

Urbina,

Phys.

Rev.

Lett.

89,

117901

(2002).

[8]

I.

Chiorescu,

Y.

Nakamura,

C.

J.

P.

M.

Harmans,

and

J.

E.

Mooij,

Science

299,

1869

(2003).

4021

[9]

Y.

A.

Pashkin,

T.

Yamamoto,

O.

Astafiev,

Y.

Nakamura,

D.

V.

Averin,

and

J.

S.

Tsai,

Nature

(London)

421,

4041

823

(2003);

T.

Yamamoto,

Yu.

A.

Pashkin,

O.

Astafiev,

Y.

Nakamura,

and

J.

S.

Tsai,

ibid.,

425,

941

(2003).

4061

[10]

A.

J.

Berkley,

H.

Xu,

R.

C.

Ramos,

M.

A.

Gubrud,

F.

W.

Strauch,

P.

R.

Johnson,

J.

R.

Anderson,

A.

J.

Dragt,

C.

J.

Lobb,

and

F.

C.

Wellstood,

Science

300,

1548

(2003).

[11]

A.

Izmalkov,

M.

Grajcar,

E.

Il'ichev,

Th.

Wagner,

H.-

G.

Meyer,

A.

Yu.

Smirnov,

M.

H.

S.

Amin,

A.

M.

van

den

Brink,

and

A.

M.

Zagoskin,

Phys.

Rev.

Lett.

93,

4129

037003

(2004).

[12]

I.

Chiorescu,

P.

Bertet,

K.

Semba,

Y.

Nakamura,

C.

J.

P.

M.

Harmans,

and

J.

E.

Mooij,

Nature

(London)

431,

159

(2004).

[13]

F.

Plastina

and

G.

Falci,

Phys.

Rev.

B

67,

4166

224514

(2003).

4169

[14]

A.

Blais,

A.

M.

van

den

Brink,

and

A.

M.

Zagoskin,

4181

Phys.

Rev.

Lett.

90,

4185

127901

(2003).

[15]

X.

Zhou,

M.

Wulf,

4193

Z-

W.

Zhou,

G-

C.

Guo,

and

M.

J.

Feldman,

Phys.

Rev.

A

69,

030301R

(2004).

[16]

A.

Blais,

R-

S.

Huang,

A.

Wallraff,

S.

M.

Girvin,

and

R.

J.

Schoelkopf,

Phys.

Rev.

A

69,

062320

(2004).

[17]

A.

Wallraff,

D.

I.

Schuster,

A.

Blais,

L.

Frunzio,

R-

S.

Huang,

J.

Majer,

S.

Kumar,

S.

M.

Girvin,

and

R.

J.

Schoelkopf,

Nature

(London)

431,

162

(2004).

4262

[18]

H.

Xu,

F.

W.

Strauch,

S.

Dutta,

P.

R.

Johnson,

R.

C.

Ramos,

A.

J.

Berkley,

H.

Paik,

J.

R.

Anderson,

A.

J.

Dragt,

C.

J.

Lobb,

and

F.

C.

Wellstood,

Phys.

Rev.

Lett.

94,

027003

(2005).

[19]

A.

M.

Kadin,

“Introduction

to

superconducting

circuits,”

Wiley-

Interscience,

1999.

4313

[20]

B.

A.

Mazin,

P.

K.

Day,

H.

G.

LeDuc,

A.

Vayonakis,

and

J.

Zmuidzinas,

Proc.

4329

SPIE

4849,

4331

283

(2002).

[21]

P.

K.

Day,

H.

G.

Leduc,

B.

A.

Mazin,

A.

Vayonakis,

and

J.

Zmuidzinas,

Nature

425,

817

(2003).

[22]

N.

Rando,

P.

Videler,

A.

Peacock,

A.

van

Dordrecht,

P.

Verhoeve,

R.

Venn,

A.

C.

Wright,

and

J.

Lumley,

J.

Appl.

Phys.

77,

4099

(1995).

[23]

J.

I.

Cirac

and

P.

Zoller,

Phys.

Rev.

Lett.

74,

4091

(1995).

[24]

T.

P.

Orlando,

J.

E.

Mooij,

L.

Tian,

C.

H.

van

der

Wal,

L.

S.

Levitov,

S.

Lloyd,

and

J.

J.

Mazo,

Phys.

Rev.

B.

60,

15398

(1999).

[25]

J.

Q.

You,

C-

H.

Lam,

and

H.

Z.

Zheng,

Phys.

Rev.

B

63,

180501R

(2001).

J.

Q.

You,

J.

S.

Tsai,

and

F.

Nori,

Phys.

Rev.

Lett.

89,

4455

197902

(2002).

[26]

Notice

as

mentioned

earlier

K^2

is

not

necessarily

the

ratio

of

junction

sizes

since

magnetic

suppression

of

critical

currents

is

usually

needed.

[27]

P.

Jordan

and

E.

Wigner,

Z.

Phys.

47,

4491

631

(1928).

[28]

J.

E.

Mooij,

T.

P.

Orlando,

L.

Levitov,

L.

Tian,

C.

H.

van

der

Wal,

and

S.Lloyd,

Science

285,

1036

(1999).

[29]

T.

V.

Filippov,

S.

K.

Tolpygo,

J.

Mannik,

and

J.

E.

Lukens,

4530

IEEE

Trans.

Appl.

Supercond.

13,

1005

(2003).

[30]

R.

H.

McKenzie,

C.

J.

Hamer,

and

D.

W.

Murray,

Phys.

Rev.

B.

53,

4553

9676

(1996).

4556

[31]

R.

J.

Bursill,

R.

H.

McKenzie,

and

C.

J.

Hamer,

Phys.

Rev.

Lett.

80,

5607

(1998).

[32]

J.

K.

Freericks,

M.

Jarrell,

and

G.

D.

Mahan,

Phys.

Rev.

Lett.

77,

4588

(1996).